Production & Industrial Engineering

Operations Research & Operations Management

Comprehensive Theory
with Solved Examples *and* Practice Questions

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CHAPTER 01

Linear Programming and Its Applications

INTRODUCTION

Linear Programming is one of the most versatile, powerful and useful techniques for making managerial decisions. Linear programming technique may be used for solving broad range of problems arising in business, government, industry, hospitals, libraries, etc. Whenever we want to allocate the available limited resources for various competing activities for achieving our desired objective, the technique that helps us is Linear Programming. As a decision making tool, it has demonstrated its value in various fields such as production, finance, marketing, research and development and personnel management. Determination of optimal product mix (a combination of products, which gives maximum profit), transportation schedules, Assignment problem and many more. In this chapter, let us discuss about various types of linear programming models.

1.1 Properties of Linear Programming Model

Any linear programming model (problem) must have the following properties:

(a) The relationship between variables and constraints must be linear.

(b) The model must have an objective function.

(c) The model must have structural constraints.

(d) The model must have non-negativity constraint.

1.2 Standard Form of LPP

A general mathematical way of representing a Linear Programming Problem (L.P.P.) is as given below :

$Z = c_1x_1 + c_2x_2 + \ldots\ldots c_nx_n$ subjects to the conditions, ⟶ OBJECTIVE FUNCTION

$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + \ldots\ldots + a_{ij}x_j + \ldots\ldots + \ldots\ldots a_{1n}x_n \; (\geq, =, \leq) \; b_1$

$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + \ldots\ldots\ldots\ldots + a_{2j}x_j + \ldots\ldots + a_{2n}x_n \; (\geq, =, \leq) \; b_2$

....

....

$a_{m1}x_1 + a_{m2}x_2 + a_{m3}x_3 + \ldots\ldots + a_{mj}x_j \ldots\ldots + a_{mn}x_n \; (\geq, =, \leq) \; b_m$

Structural Constraints

and all x_j are = 0 ⟶ NON NEGETIVITY CONSTRAINT

where $j = 1, 2, 3, \ldots\ldots, n$

where,

$$x_1 \geq 0, x_2 \geq 0 \ldots x_n \geq 0, x_{n+1} \geq 0 \ldots x_{n+m} \geq 0$$

where $x_1, x_2, \ldots x_n$ are called decision variables.

$x_{n+1}, x_{n+2} \dots x_{n+m}$
are called slack variable.

$c_1, c_2, \dots\dots c_n$ are called cost factors.

- Coefficient of slack variables $x_{n+1}, x_{n+2} \dots x_{n+m}$ in the objective function are assumed to be zero.
- Since in case of (≥ 0) constraints, the subtracted variables represents the surplus of the left side over the right side, it is common to refer to it as surplus variables.
- **Solution to LPP:** Any set $x\{x_1, x_2, \dots\dots x_{n+m}\}$ of variables is called as a solution to LPP, it is satisfy the set of constraints (2) only.

The steps for formulating the linear programming are:

1. Identify the unknown decision variables to be determined and assign symbols to them.
2. Identify all the restrictions or constraints in the problem and express them as linear equations or inequalities of decision variables.
3. Identify the objective or aim and represent it also as a linear function of decision variables.

1.3 Basic Assumptions

The following are some important assumptions made in formulating a linear programming model :

1. It is assumed that the decision maker here is completely certain (i.e., deterministic conditions) regarding all aspects of the situation, i.e., availability of resources, profit contribution of the products, technology, courses of action and their consequences etc.
2. It is assumed that the relationship between variables in the problem and the resources available i.e., constraints of the problem exhibits linearity. Here the term linearity implies proportionality and additivity. This assumption is very useful as it simplifies modeling of the problem.
3. We assume here fixed technology. Fixed technology refers to the fact that the production requirements are fixed during the planning period and will not change in the period.
4. It is assumed that the profit contribution of a product remains constant, irrespective of level of production and sales.
5. It is assumed that the decision variables are continuous. It means that the companies manufacture products in fractional units. For example, company manufacture 2.5 vehicles, 3.2 barrels of oil etc. This is referred too as the assumption of divisibility.
6. It is assumed that only one decision is required for the planning period. This condition shows that the linear programming model is a static model, which implies that the linear programming problem is a single stage decision problem. (Note: Dynamic Programming problem is a multistage decision problem).
7. All variables are restricted to nonnegative values (i.e., their numerical value will be ≥ 0).

1.4 Steps to Formulate LPP Model

The steps for formulating the linear programming are:

1. Identify the unknown decision variables to be determined and assign symbols to them.
2. Identify all the restrictions or constraints in the problem and express them as linear equations or inequalities of decision variables.
3. Identify the objective or aim and represent it also as a linear function of decision variables.

We can further categories the problem into maximization and minimization model.

1.4.1 Maximization Model

Herein, our aim is to maximize the objective function with all the variables subjected to given constraint. Following example illustrates maximization model of a linear programming problem.

Example 1.1 A retail store stocks two types of shirts *A* and *B*. These are packed in attractive cardboard boxes. During a week the store can sell a maximum of 400 shirts of type *A* and a maximum of 300 shirts of type *B*. The storage capacity, however, is limited to a maximum of 600 of both types combined. Type *A* shirt fetches a profit of Rs. 2/- per unit and type *B* a profit of Rs. 5/- per unit. How many of each type the store should stock per week to maximize the total profit? Formulate a mathematical model of the problem.

Solution:

Here shirts *A* and *B* are problem variables. Let the store stock '*a*' units of *A* and '*b*' units of *B*. As the profit contribution of *A* and *B* are Rs. 2/- and Rs. 5/- respectively, objective function is: Maximize $Z = 2a + 5b$ subjected to condition (s.t.)

Structural constraints are, stores can sell 400 units of shirt *A* and 300 units of shirt *B* and the storage capacity of both put together is 600 units. Hence the structural constraints are: $1a + 0b \geq 400$ and $0a + 1b \leq 300$ for sales capacity and $1a + 1b \leq 600$ for storage capacity. And non-negativity constraint is both *a* and *b* are ≥ 0. Hence the model is:

$$\text{Maximize } Z = 2a + 5b \text{ s.t.}$$
$$1a + 0b \leq 400$$
$$0a + 1b \leq 300$$
$$1a + 1b \leq 600 \text{ and both } a \text{ and } b \text{ are} \geq 0.$$

1.4.2 Minimization Model

Herein, our aim is to maximize the objective function with all the variables subjected to given constraint. Following example illustrates maximization model of a linear programming problem.

Example 1.2 A patient consult a doctor to check up his ill health. Doctor examines him and advises him that he is having deficiency of two vitamins, vitamin A and vitamin D. Doctor advises him to consume vitamin A and D regularly for a period of time so that he can regain his health. Doctor prescribes tonic X and tonic Y, which are having vitamin A, and D in certain proportion. Also advises the patient to consume at least 40 units of vitamin A and 50 units of vitamin Daily. The cost of tonics *X* and *Y* and the proportion of vitamin A and D that present in *X* and *Y* are given in the table below. Formulate l.p.p. to minimize the cost of tonics.

Vitamins	Tonics		Daily requirement in units
	X	*Y*	
A	2	4	40
D	3	2	50
Cost in Rs. per unit	5	3	

Solution:

Let patient purchase x units of *X* and y units of *Y*.

Objective function: Minimize $Z = 5x + 3y$

Inequality for vitamin A is $2x + 4y \geq 40$ (Here at least word indicates that the patient can consume more than 40 units but not less than 40 units of vitamin A daily).

Similarly, the inequality for vitamin *D* is $3x + 2y \geq 50$.

For non-negativity constraint the patient cannot consume negative units. Hence both x and y must be ≥ 0.

Now the l.p.p. model for the problem is :

$$\text{Minimize } Z = 5x + 3y \text{ s.t.}$$
$$2x + 4y \geq 40$$
$$3x + 2y \geq 50 \text{ and}$$

Both x and y are ≥ 0.

1.5 Methods for the Solution of Linear Programming Problem

1. The Graphical Method when we have two decision variables in the problem. (To deal with more decision variables by graphical method will become complicated, because we have to deal with planes instead of straight lines. Hence in graphical method let us limit ourselves to two variable problems.
2. The Systematic Trial and Error method, where we go on giving various values to variables until we get optimal solution. This method takes too much of time and laborious, hence this method is not discussed here.
3. The Vector method. In this method each decision variable is considered as a vector and principles of vector algebra is used to get the optimal solution. This method is also time consuming, hence it is not discussed here.
4. The Simplex method. When the problem is having more than two decision variables, simplex method is the most powerful method to solve the problem. It has a systematic program, which can be used to solve the problem.

NOTE : One problem with two variables is solved by using both graphical and simplex method.

1.5.1 Graphical Method

So far we have learnt how to construct a mathematical model for a linear programming problem. If we can find the values of the decision variables x_1, x_2, x_3, ... x_n, which can optimize (maximize or minimize) the objective function Z, then we say that these values of x_i are the optimal solution of the Linear Program (LP).

The graphical method is applicable to solve the LPP involving two decision variables x_1 and x_2, we usually take these decision variables as x, y instead of x_1, x_2.

The characteristics of Graphical method are:

(i) Generally the method is used to solve the problem, when it involves two decision variables.

(ii) For three or more decision variables, the graph deals with planes and requires high imagination to identify the solution area.

(iii) Always, the solution to the problem lies in first quadrant.

(iv) This method provides a basis for understanding the other methods of solution.

To solve an LP, the graphical method includes two major steps.

(a) The determination of the solution space that defines the feasible solution. Note that the set of values of the variable x_1, x_2, x_3, ... x_n which satisfy all the constraints and also the non-negative conditions is called the feasible solution of the LP.

(b) The determination of the optimal solution from the feasible region.

(A) Steps to determine feasible solution

Step 1: Since the two decision variable x and y are non-negative, consider only the first quadrant of xy-coordinate plane.

Step 2: Each constraint is of the form $ax + by \leq c$ or $ax + by \geq c$.

Draw the line $\quad ax + by = c \quad$ ...(1)

For each constraint, the line (1) divides the first quadrant into two regions say R_1 and R_2, suppose (x_1, 0) is a point in R_1. If this point satisfies the in equation $ax + by$ ... c or (...., c), then shade the region R_1. If (x_1, 0) does not satisfy the inequality, shade the region R_2.

Step 3: Corresponding to each constant, we obtain a shaded region. The intersection of all these shaded regions is the feasible region or feasible solution of the LP.

Example 1.3 Find the feasible solution for the problem of a decorative item dealer whose LPP is to maximize profit function.

$$Z = 50x + 18y \qquad \ldots(1)$$

Subject to the constraints

$$2x + y \leq 100$$
$$x + y \leq 80$$
$$x \geq 0,\ y \geq 0$$

Solution:

Step 1: Since $x \geq 0$, $y \geq 0$, we consider only the first quadrant of the xy-plane

Step 2: We draw straight lines for the equation

$$2x + y = 100 \qquad \ldots(2)$$
$$x + y = 80$$

To determine two points on the straight line $2x + y = 100$

Put $y = 0$, $2x = 100$

$\Rightarrow$ $x = 50$

$\Rightarrow$ (50, 0) is a point on the line (2)

Put $x = 0$ in (2), $y = 100$

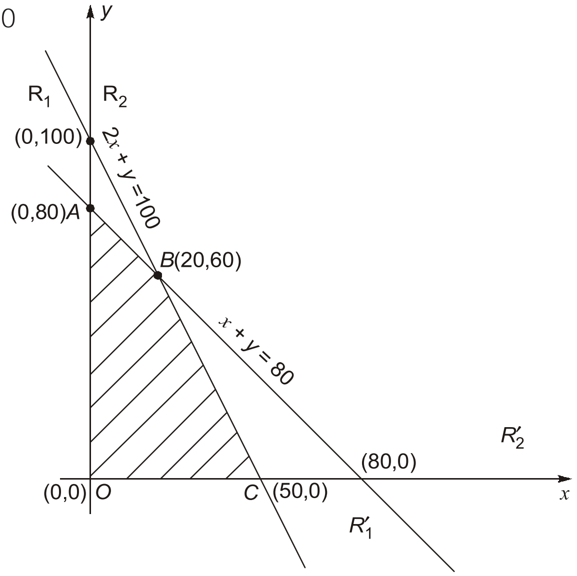
$\Rightarrow$ (0, 100) is the other point on the line (2)

Plotting these two points on the graph paper draw the line which represent the line $2x + y = 100$.

This line divides the 1st quadrant into two regions, say R_1 and R_2. Choose a point say (1, 0) in R_1. (1, 0) satisfy the inequality $2x + y \leq 100$. Therefore R_1 is the required region for the constraint

$$2x + y \leq 100$$

Similarly draw the straight line $x + y = 80$ by joining the point (0, 80) and (80, 0). Find the required region say R'_1, for the constraint $x + y \leq 80$.



The intersection of both the region R_1 and R'_1 is the feasible solution of the LPP. Therefore every point in the shaded region OABC is a feasible solution of the LPP, since this point satisfies all the constraints including the non-negative constraints.

(B) Steps to find optimal solution of an LPP

There are two techniques to find the optimal solution of an LPP :

(i) Corner point method

(ii) Iso-Profit or Iso-Cost method

(i) Corner point method : The optimal solution to a LPP, if it exists, occurs at the corners of the feasible region. The method includes the following steps

Step 1: Find the feasible region of the LLP.

Step 2: Find the co-ordinates of each vertex of the feasible region.

These co-ordinates can be obtained from the graph or by solving the equation of the lines.

Step 3: At each vertex (corner point) compute the value of the objective function.

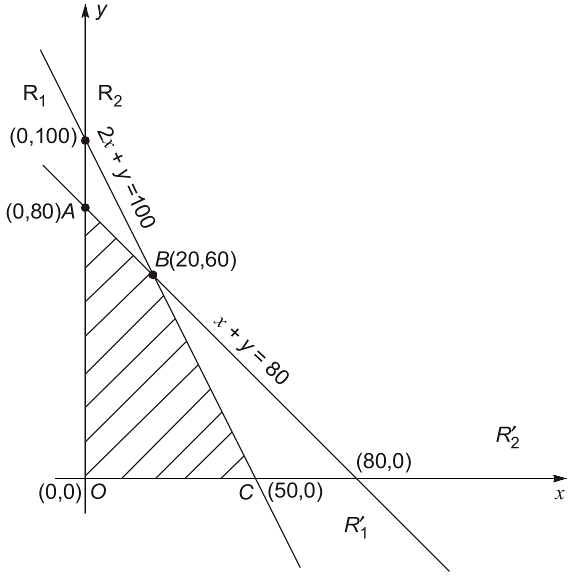
Step 4: Identify the corner point at which the value of the objective function is maximum (or minimum depending on the LP)

The co-ordinates of this vertex is the optimal solution and the value of Z is the optimal value.

Example 1.4 Find the optimal solution for the problem 1.3 (of a decorative item dealer) whose LPP is to maximize profit function.

$$Z = 50x + 18y$$

Solution:



In the graph, the corners of the feasible region are:

$O(0, 0)$, $A(0, 80)$, $B(20, 60)$, $C(50, 0)$

- At (0, 0) $Z = 0$
- At (0, 80) $Z = 50(0) + 18(80) = 1440$
- At (20, 60) $Z = 50(20) + 18(60) = 1000 + 1080 = \text{Rs.}2080$
- At (50, 0) $Z = 50(50) + 18(0) = 2500$

Since our object is to maximize Z and Z has maximum at (50, 0) the optimal solution is

$$x = 50 \text{ and } y = 0.$$

The optimal value is 2500.

(ii) Iso-Profit or Iso-Cost method : If an LPP has many constraints, then it may be long and tedious to find all the corners of the feasible region. There is another alternate and more general method to find the optimal solution of an LP, known as 'ISO profit or ISO cost method'.

Suppose the LPP is to optimize $Z = ax + by$ subject to the constraints

$$a_1x + b_1y \le (\text{or} \ge) c_1$$
$$a_2x + b_2y \le (\text{or} \ge) c_2$$
$$x \ge 0,\ y \ge 0.$$

This method of optimization involves the following method.

Step 1: Draw the half planes of all the constraints.

Step 2: Shade the intersection of all the half planes which is the feasible region.

Step 3: Since the objective function is $Z = ax + by$, draw a dotted line for the equation

$$ax + by = k,$$

where k is any constant. Sometimes it is convenient to take k as the LCM of a and b.

Step 4: To maximize Z draw a line parallel to $ax + by = k$ and farthest from the origin. This line should contain at least one point of the feasible region. Find the coordinates of this point by solving the equations of the lines on which it lies.

To minimize Z draw a line parallel to $ax + by = k$ and nearest to the origin. This line should contain at least one point of the feasible region. Find the co-ordinates of this point by solving the equation of the line on which it lies.

Step 5: If (x_1, y_1) is the point found in step 4, then $x = x_1$, $y = y_1$, is the optimal solution of the LPP and $Z = ax_1 + by_1$ is the optimal value.

Example 1.5 Solve the following LPP graphically using ISO- profit method.

Objective function: maximize $Z = 100 + 100y$.

Subject to the constraints:

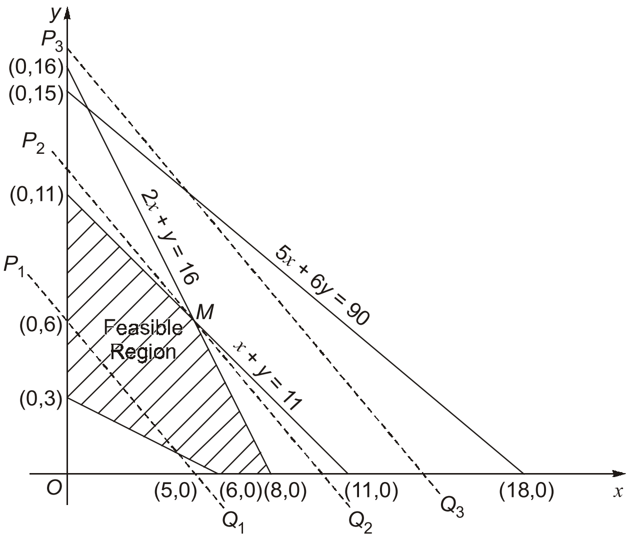
$$10x + 5y \leq 80$$
$$6x + 6y \leq 66$$
$$4x + 8y \geq 24$$
$$5x + 6y \leq 90$$
$$x \geq 0,\ y \geq 0$$

Solution:

Since $x \geq 0$, $y \geq 0$, consider only the first quadrant of the plane graph the following straight lines on a graph paper

$$10x + 5y = 80 \text{ or } 2x + y = 16$$
$$6x + 6y = 66 \text{ or } x + y = 11$$
$$4x + 8y = 24 \text{ or } x + 2y = 6$$
$$5x + 6y = 90$$

Identify all the half planes of the constraints. The intersection of all these half planes is the feasible region as shown in the figure.



Give a constant value 600 to Z in the objective function, then we have an equation of the line

$$120x + 100y = 600 \quad \text{...(1)}$$
$$6x + 5y = 30 \text{ (Dividing both sides by 20)}$$

P_1Q_1 is the line corresponding to the equation $6x + 5y = 30$. We give a constant 1200 to Z then the P_2Q_2 represents the line.

$$120x + 100y = 1200$$
$$6x + 5y = 60$$

P_2Q_2 is a line parallel to P_1Q_1 and has one point 'M' which belongs to feasible region and farthest from the origin. If we take any line P_3Q_3 parallel to P_2Q_2 away from the origin, it does not touch any point of the feasible region.

The co-ordinates of the point M can be obtained by solving the equation $2x + y = 16$

$$x + y = 11 \text{ which give}$$

$$x = 5 \text{ and } y = 6$$

$\Rightarrow$ The optimal solution for the objective function is $x = 5$ and $y = 6$

The optimal value of Z : 120 (5) + 100 (6) = 600 + 600 = 1200

Points to be Noted in Graphical Method:

(i) In case Iso-profit line passes through more than one point, then it means that the problem has more than one optimal solution, i.e., alternate solutions all giving the same profit. This helps the manager to take a particular solution depending on the demand position in the market. He has options.

(ii) If the Iso-profit line passes through single point, it means to say that the problem has unique solution (fig a).

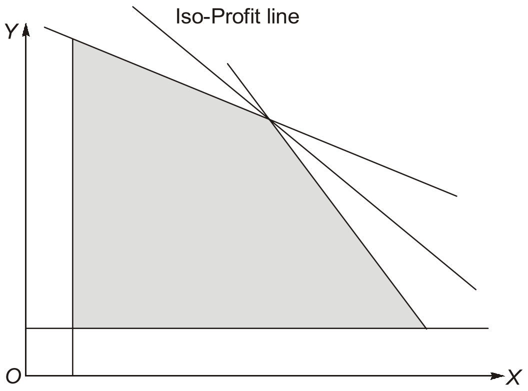


Fig (a): Unique optimal solution

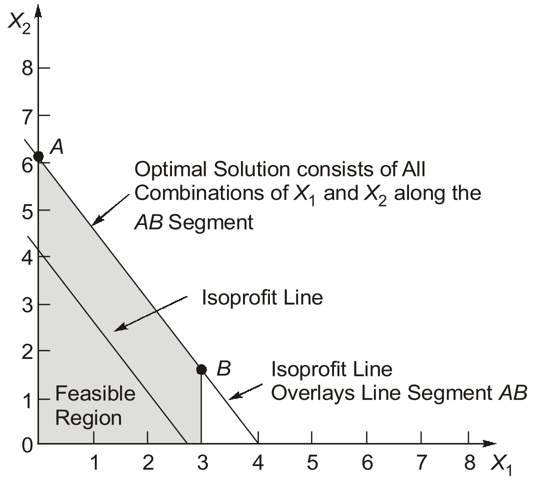


Fig (b): Innumerable or infinite solutions

(iii) If the Iso-profit line coincides any one line of the polygon, then all the points on the line are solutions, yielding the same profit. Hence the problem has innumerable solutions (fig b).

(iv) If the line do not pass through any point (in case of open polygons), then the problem do not have solution, and we say that the problem is **Unbound** (fig c).

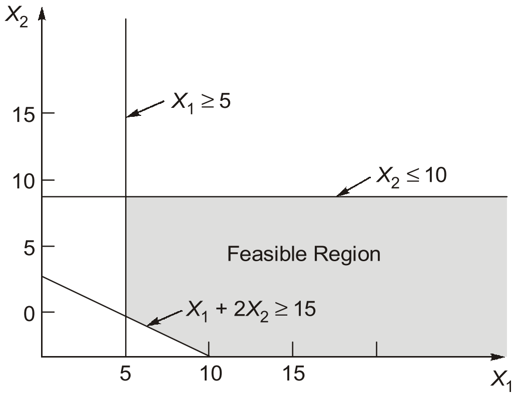


Fig (c): Unbounded solution

Example 1.6 Formulate the l.p.p. and solve the below given problem graphically. Old hens can be bought for Rs.2.00 each but young ones costs Rs. 5.00 each. The old hens lay 3 eggs per week and the young ones lay 5 eggs per week. Each egg costs Rs. 0.30. A hen costs Rs. 1.00 per week to feed. If the financial constraint is to spend Rs. 80.00 per week for hens and the capacity constraint is that total number of hens cannot exceed 20 hens and the objective is to earn a profit more than Rs. 6.00 per week, find the optimal combination of hens.

Solution:

Let x be the number of old hens and y be the number of young hens to be bought. Now the old hens lay 3 eggs and the young one lays 5 eggs per week. Hence total number of eggs one get is

$$3x + 5y$$

Total revenues from the sale of eggs per week is Rs. $0.30(3x + 5y)$ i.e., $0.90x + 1.5y$.

Now the total expenses per week for feeding hens is Re. $1(1x + 1y)$ i.e., $1x + 1y$.

Hence the net income :

$$\text{Revenue} - \text{Cost} = (0.90x + 1.5y) - (1x + 1y) = -0.1x + 0.5y \text{ or } 0.5y - 0.1x.$$

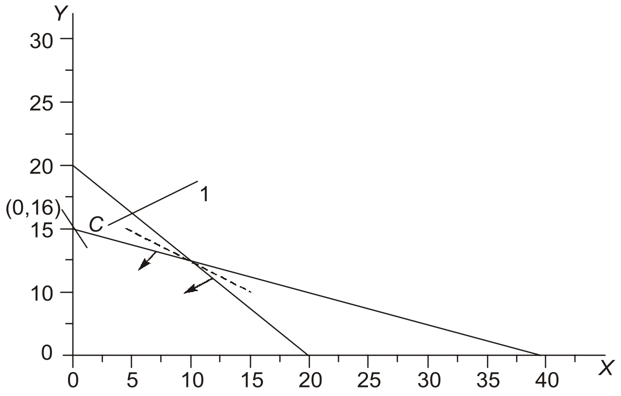
Hence the desired l.p.p. is

$$\text{Maximise } Z = 0.5y - 0.1 \times \text{ S.T.}$$
$$2x + 5y \le 80$$
$$1x + 1y \le 20 \text{ and both } x \text{ and } y \text{ are} \ge 0.$$

The equations are:

$$\text{Maximise } Z = 0.5y - 0.1 \times \text{ S.T.}$$
$$2x + 5y = 80$$
$$1x + 1y = 20 \text{ and both } x \text{ and } y \text{ are} \ge 0.$$

In the below figure, which shows the graph for the problem, the isoprofit line passes through the point C. Hence $Z_c = Z(0, 16)$ = Rs. 8.00. Hence, one has to buy 16 young hens and his weekly profit will be Rs.8.00.



Example 1.7 A company manufactures two products X and Y. The profit contribution of X and Y are Rs. 3/- and Rs. 4/- respectively. The products X and Y require the services of four facilities. The capacities of the four facilities A, B, C and D are limited and the available capacities in hours are 200 Hrs, 150 Hrs, and 100 Hrs. and 80 hours respectively. Product X requires 5, 3, 5 and 8 hours of facilities A, B, C and D respectively. Similarly the requirement of product Y is 4, 5, 5, and 4 hours respectively on A, B, C and D. Find the optimal product mix to maximise the profit.

Solution:

Machines	Products		Availability in hours
	X (Time in hours)	*Y*	
A	5	4	200
B	3	5	150
C	5	4	100
D	8	4	80
Profit in Rs. Per unit :	3	4	

The inequalities and equations for the above data will be as follows. Let the company manufactures x units of X and y units of Y.

Maximise $Z = 3x + 4y$ S.T.

$5x + 4y \le 200$

$3x + 5y \le 150$

$5x + 4y \le 100$

$8x + 4y \le 80$

Maximise $Z = 3x + 4y$ S.T.

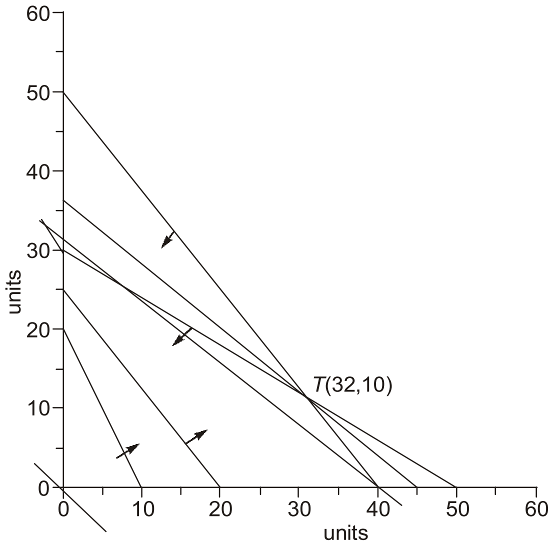
$5x + 4y = 200$

$3x + 5y = 150$

$5x + 4y = 100$

$8x + 4y = 80$

And both x and y are ≥ 0 and both x and y are ≥ 0.



In the graph the line representing the equation $8x + 4y$ is outside the feasible area and hence it is a redundant equation. It does not affect the solution. The Isoprofit line passes through corner T of the polygon and is the point of maximum profit. Therefore $ZT = Z(32, 10) = 3 \times 32 + 4 \times 10 =$ Rs. 136/.

1.5.2 Simplex Method

The simplex method is based on the property that the optimal solution to a linear programming problem, if it exists, can always be found in one of the basic feasible solution. The simplex method is quite simple and mechanical in nature. The iterative steps of the simplex method are repeated until a finite optimal solution, if exists, is found. If no optimal solution, the method indicates that no finite solution exists.

Comparison between Graphical and Simplex Method

1. The graphical method is used when we have two decision variables in the problem. Whereas in Simplex method, the problem may have any number of decision variables.
2. In graphical method, the inequalities are assumed to be equations, so as to enable to draw straight lines. But in Simplex method, the inequalities are converted into equations by:

(i) Adding a SLACK VARIABLE in maximisation problem

(ii) Subtracting a SURPLUS VARIABLE in case of minimisation problem.

3. In graphical solution the Isoprofit line moves away from the origin to towards the far off point in maximisation problem and in minimisation problem, the Isocost line moves from far off distance towards origin to reach the nearest point to origin.
4. In graphical method, the areas outside the feasible area (area covered by all the lines of constraints in the problem) indicates idle capacity of resource where as in Simplex method, the presence of slack variable indicates the idle capacity of the resources.
5. In graphical solution, if the isoprofit line coincides with more than one point of the feasible polygon, then the problem has second alternate solution. In case of Simplex method the net-evaluation row has zero for non-basis variable the problem has alternate solution. (If two alternative optimum solutions can be obtained, the infinite number of optimum, solutions can be obtained).

Steps for Simplex Method for Maximization Problem

Following are the major steps to be followed for simplex method:

(1) Standard form
(2) Introducing slack variables
(3) Creating the tableau
(4) Pivot variables
(5) Creating a new tableau
(6) Checking for optimality
(7) Identify optimal values

These steps are discussed in detail using the following example :

Objective function:

$$\text{Maximize } Z = 8x_1 + 10x_2 + 7x_3$$

Constraints:

$$x_1 + 5x_2 + x_3 \leq 8$$
$$x_1 + 3x_2 + 2x_3 \leq 10$$
$$x_1, x_2, x_3 \geq 0$$

Steps 1: Standard form : Standard form is the baseline format for all linear programs before solving for the optimal solution and has three requirements :

(1) must be a maximization problem,
(2) all linear constraints must be in a less-than-or-equal-to inequality,
(3) all variables are non-negative.

These requirements can always be satisfied by transforming any given linear program using basic algebra and substitution. Standard form is necessary because it creates an ideal starting point for solving the Simplex method as efficiently as possible as well as other methods of solving optimization problems.

Step 2: Determine Slack Variables : Slack variables are additional variables that are introduced into the linear constraints of a linear program to transform them from inequality constraints to equality constraints. If the model is in standard form, the slack variables will always have *a* +1 coefficient. Slack variables are needed in the constraints to transform them into solvable equalities with one definite answer.

$$x_1 + 3x_2 + 2x_3 + x_1 = 10$$
$$x_1 + 5x_2 + x_3 + s_2 = 8$$
$$x_1, x_2, x_3, s_1, s_2 \geq 0$$

Step 3: Setting up the table : A Simplex table is used to perform row operations on the linear programming model as well as to check a solution for optimality. The table consists of the coefficient corresponding to the linear constraint variables and the coefficients of the objective function. In the table below, the bolded top row of the table states what each column represents. The following two rows represent the linear constraint variable coefficients from the linear programming model, and the last row represents the objective function variable coefficients.

$$\text{Maximize}: z = 8x_1 + 10x_2 + 7x_3$$

$$\text{s.t.}: x_1 + 3x_2 + 2x_3 + s_1 = 10$$

$$x_1 + 5x_2 + x_3 + s_2 = 8$$

x_1	x_2	x_3	s_1	s_2	z	b
1	3	2	1	0	0	10
1	5	1	0	1	0	8
–8	–10	–7	0	0	1	0

Step 4: Check Optimality : The optimal solution of a maximization linear programming model are the values assigned to the variables in the objective function to give the largest zeta value. The optimal solution would exist on the corner points of the graph of the entire model. To check optimality using the table, all values in the last row must contain values greater than or equal to zero. If a value is less than zero, it means that variable has not reached its optimal value. As seen in the previous table, three negative values exists in the bottom row indicating that this solution is not optimal. If a table is not optimal, the next step is to identify the pivot variable to base a new table on, as described in Step 5.

Step 5: Identify Pivot Variable : The pivot variable is used in row operations to identify which variable will become the unit value and is a key factor in the conversion of the unit value. The pivot variable can be identified by looking at the bottom row of the tableau and the indicator. Assuming that the solution is not optimal, pick the smallest negative value in the bottom row. One of the values lying in the column of this value will be the pivot variable. To find the indicator, divide the beta values of the linear constraints by their corresponding values from the column containing the possible pivot variable. The intersection of the row with the smallest non-negative indicator and the smallest negative value in the bottom row will become the pivot variable.

In the example shown below, –10 is the smallest negative in the last row. This will designate the x_2 column to contain the pivot variable. Solving for the indicator gives us a value of $\frac{10}{3}$ for the first constraint, and a value of $\frac{8}{5}$ for the second constraint. Due to $\frac{8}{5}$ being the smallest non-negative indicator, the pivot value will be in the second row and have a value of 5.

x_1	x_2	x_3	s_1	s_2	z	b	Indicator
1	3	2	1	0	0	10	10/3
1	⑤	1	0	1	0	8	8/5
–8	–10	–7	0	0	1	0	

↑ Smallest Value

Now that the new pivot variable has been identified, the new table can be created in Step 6 to optimize the variable and find the new possible optimal solution.

Step 6: Create the New Table : The new table will be used to identify a new possible optimal solution. Now that the pivot variable has been identified in Step 5, row operations can be performed to optimize the pivot variable while keeping the rest of the table equivalent.

I. To optimize the pivot variable, it will need to be transformed into a unit value (value of 1). To transform the value, multiply the row containing the pivot variable by the reciprocal of the pivot value. In the example below, the pivot variable is originally 5, so multiply the entire row by $\frac{1}{5}$.

x_1	x_2	x_3	s_1	s_2	z	b
1/5	①	1/5	0	1/5	0	8/5

Pivot row

II. After the unit value has been determined, the other values in the column containing the unit value will become zero. This is because the x_2 in the second constraint is being optimized, which requires x_2 in the other equations to be zero.

x_1	x_2	x_3	s_1	s_2	z	b
	0					
1/5	①	1/5	0	1/5	0	8/5
	0					

Pivot row

Pivot Column

III. In order to keep the table equivalent, the other variables not contained in the pivot column or pivot row must be calculated by using the new pivot values. For each new value, multiply the negative of the value in the old pivot column by the value in the new pivot row that corresponds to the value being calculated. Then add this to the old value from the old table to produce the new value for the new table. This step can be condensed into the equation as:

New tableau value = (Negative value in old tableau pivot column) × (value in new tableau pivot row) + (Old tableau value)

Old table

x_1	x_2	x_3	s_1	s_2	z	b
1	3	2	1	0	0	10
1	⑤	1	0	1	0	8
–8	–10	–7	0	0	1	0

Old pivot column

New table

x_1	x_2	x_3	s_1	s_2	z	b
2/5	0	7/5	1	–3/5	0	26/5
1/5	①	1/5	0	1/5	0	8/5
–6	0	–5	0	2	1	16

New pivot row

Step 7: Check Optimality : As explained in Step 4, the optimal solution of a maximization linear programming model are the values assigned to the variables in the objective function to give the largest zeta value. Optimality will need to be checked after each new table to see if a new pivot variable needs to be identified. A solution is considered optimal if all values in the bottom row are greater than or equal to zero. If all values are greater than or equal to zero, the solution is considered

Again it is not optimal. Again performing the same sequence of steps to get optimal one.

Casting	Machines			
	M_1	M_2	M_3	M_4
c_1	$\boxed{0}$	$\cancel{0}$	$\cancel{0}$	$\cancel{0}$
c_2	10	$\boxed{0}$	$\cancel{0}$	4
c_3	$\cancel{0}$	2	$\boxed{0}$	4
c_4	16	$\cancel{0}$	$\cancel{0}$	$\boxed{0}$

As there are more than one zero in each row and column, multiple assignments are possible.

$$\begin{bmatrix} C_1 \to M_1 \\ C_2 \to M_2 \\ C_3 \to M_3 \\ C_4 \to M_4 \end{bmatrix}$$

$$\text{Solution (cost)} = 10 + 10 + 20 + 6 = 46$$

or other solution,

$$\begin{bmatrix} C_1 \to M_3 \\ C_2 \to M_2 \\ C_3 \to M_1 \\ C_4 \to M_4 \end{bmatrix}$$

$$\text{Cost} = 8 + 10 + 22 + 6 = 46.$$

Student's Assignments 1

Q.1 The technique of Dynamic Programming problem is developed by:
(a) Taylor (b) Gilberth
(c) Richard Bellman (d) Bellman and Clarke

Q.2 Another name used to Dynamic Programming is:
(a) Multistage problem
(b) Recursive optimization
(c) State problems
(d) No second name

Q.3 If the outcome at any decision stage is unique and known for the problem, then the Dynamic programming problem is known as:
(a) Probabilistic dynamic programming problem
(b) Stochastic dynamic programming problem
(c) Static dynamic programming problem
(d) Deterministic dynamic programming problem

Q.4 Transportation problem is basically a
(a) Maximization model
(b) Minimization model
(c) Transshipment problem
(d) Iconic model

Q.5 The column, which is introduced in the matrix to balance the rim requirements, is known as:
(a) Key column (b) Idle column
(c) Slack column (d) Dummy column

Q.6 The row, which is introduced in the matrix to balance the rim requirement, is known as:
(a) Key row (b) Idle row
(c) Dummy row (d) Slack row

Q.7 One of the differences between the Resource allocation model and Transportation Model is:
(a) The coefficients of problem variables in Resource allocation model may be any number and in transportation model it must be either zeros or ones.
(b) The coefficients of problem variable in Resource allocation model must be either zeros or ones and in Transportation model they may be any number.
(c) In both models they must be either zeros or ones only.
(d) In both models they may be any number.

Q.8 To convert the transportation problem into a maximization model we have to

(a) To write the inverse of the matrix
(b) To multiply the rim requirements by –1
(c) To multiply the matrix by –1
(d) We cannot convert the transportation problem in to a maximization problem, as it is basically a minimization problem

Q.9 In a transportation problem where the demand or requirement is equals to the available resource is known as
(a) Balanced transportation problem,
(b) Regular transportation problem,
(c) Resource allocation transportation problem
(d) Simple transportation model.

Q.10 The total number of allocation in a basic feasible solution of transportation problem of $m \times n$ size is equal to:
(a) $m \times n$
(b) $(m/n) - 1$
(c) $m + n + 1$
(d) $m + n - 1$

Q.11 When the total allocations in a transportation model of $m \times n$ size is not equals to $m + n - 1$ the situation is known as:
(a) Unbalanced situation
(b) Tie situation
(c) Degeneracy
(d) None of the above

Q.12 VAM stands for :
(a) Value added method
(b) Value assessment method
(c) Vogel Adam method,
(d) Vogel's approximation method.

Q.13 MODI stands for
(a) Modern distribution,
(b) Mendel's distribution method
(c) Modified distribution method
(d) Model index method

Q.14 In transportation model, the opportunity cost is given by
(a) Implied cost + Actual cost of the cell
(b) Actual cost of the cell – Implied cost
(c) Implied cost – Actual cost of the cell
(d) Implied cost × Actual cost of the cell

Q.15 In Hungarian method of solving assignment problem, the row opportunity cost matrix is obtained by:
(a) Dividing each row by the elements of the row above it,
(b) By subtracting the elements of the row from the elements of the row above it.
(c) By subtracting the smallest element from all other elements of the row.
(d) By subtracting all the elements of the row from the highest element in the matrix

Q.16 The horizontal and vertical lines drawn to cover all zeros of total opportunity matrix must be:
(a) Equal to each other,
(b) Must be equal to $m \times n$ (where m and n are number of rows and columns)
(c) $m + n$(m and n are number of rows and columns)
(d) Number of rows or columns

Q.17 When we try to solve assignment problem by transportation algorithm the following difficulty arises:
(a) There will be a tie while making allocations
(b) The problem will get alternate solutions,
(c) The problem degenerate and we have to use epsilon to solve degeneracy
(d) We cannot solve the assignment problem by transportation algorithm.

Q.18 Which of the following law is the not used in linear programming?
(a) Law of proportionality
(b) Law of divisibility
(c) Law of certainty
(d) Law of subtraction

Q.19 When slope of binding variable becomes equal to the slope of objective function?
(a) There exist a unique solution
(b) There exist infinite number of solution
(c) There exist no solution
(d) None of the above

Q.20 In an linear programming problem, number of variable is 5 and number of constraint is 3, then the number of solutions is ________.

Q.21 In a transportation problem, there are 5 supply points and 4 demand points. Total demand exceeds total supply by 10 units, then the minimum number of allocation required is ____.

sales representative for different sale territories are shown in the following table. The total sales increase per month is ________.

Sales representative	Sales territories			
	I	II	III	IV
A	200	150	170	220
B	160	120	150	140
C	190	195	190	200
D	180	175	160	190

Q.62 Consider the following transportation model, the minimum number of units allocated from plant *C* to warehouse *F* using north west corner rule is __.

		Warehouse			
		D	*E*	*F*	Supply
Plant	*A*	6	4	1	50
	B	3	8	7	40
	C	4	4	2	60
Demand		20	95	35	

ANSWERS

1. (c)	2. (b)	3. (d)	4. (b)	5. (d)
6. (d)	7. (c)	8. (a)	9. (c)	10. (d)
11. (c)	12. (d)	13. (c)	14. (b)	15. (c)
16. (d)	17. (c)	18. (d)	19. (b)	20. (10)
21. (9)	22. (d)	23. (b)	24. (a)	25. (c)
26. (c)	27. (a)	28. (c)	29. (c)	30. (b)
31. (d)	32. (b)	33. (c)	34. (d)	35. (30)
36. (1664)	37. (26)	38. (75)	39. (a)	40. (d)
41. (39)	42. (d)	43. (d)	44. (55)	45. (a)
46. (–6.667)	47. (d)	48. (1493)	49. (1300)	50. (110)
51. (d)	52. (b)	53. (c)	54. (c)	55. (1)
56. (1.42553)	57. (189)	58. (4)	59. (a)	60. (a)
61. (745)	62. (35)			

HINTS

19. (b)
When slope of binding variable becomes equal to the slope of objective function then there exists infinite number of solutions.

20. (10)
Number of variable,
$$n = 5$$
Number of constraint,
$$m = 3$$
Number of solution
$$= {}^nC_m = {}^5C_3 = 10$$

21. (9)
Demand exceeds supply by 10, so we have to make additional dummy supply points.
So, number of supply points = 6,
Demand points = 4
So, Minimum number of allocation
$$= m + n - 1 = 6 + 4 - 1 = 9$$

22. (d)
If all the values in the replacement ratio column are either negative or infinite then the solution terminates and it is the case of unbounded solution.

23. (b)
When slope of objective function is not equal to any of constraints it gives unique solution.

25. (c)
Coefficient of slack variable is zero in objective function.

26. (c)
It is always equal to zero.

27. (a)
$\Sigma a_i \neq \Sigma b_j$ (Supply and demand) problem is unbalanced.

28. (c)
One or more basic variables are zero in degenerate solution.

32. (b)
Objective function coefficient for i^{th} variable in dual.

34. (d)
For performing optimality test initial solution must be non-degenerate. For non-degenerate solution total number of allocations must be exactly equal to $(m + n - 1)$ and all these allocations must be at independent positions.